GAME THEORY AND EQUILIBRIA

A short primer on terminology

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Introduction

The following text summarises the key concepts of game theory and equilibria, for use by economists, as succinctly as possible. It is not intended to provide an introduction to those unfamiliar with the topics, for which purpose a standard undergraduate text such as David Kreps's *A Course in Microeconomic Theory* is recommended.

In game theory, we are usually looking for one or more equilibria (ideally only one), which we regard as representing the likely outcome of a particular situation. The principal criteria which an equilibrium is expected to satisfy are the Nash equilibrium condition and 'subgame perfection'. Before we can define either of these criteria, we need to define the concept of 'dominant strategy'.

Dominant strategies

We can write the strategy of player *i* as s_i. By strategy we mean a particular move or policy, e.g. "produce low output, rather than high output", or "cooperate if and only if the other player did so in the previous round".

We can write the payoff to player 1,
if player 1 plays s_1 and player 2 plays s_2,
as $u_1(s_1,s_2)$.

Write particular possible strategies which player 1 can adopt as
s_1^A, s_1^B, etc.

If $u_1(s_1^A,s_2) < u_1(s_1^B, s_2)$ for all possible s_2 and some s_1^B
(i.e. whatever player 2 does, it is possible for player 1
to do better than s_1^A)
then s_1^A is a *dominated* strategy.

If a strategy remains after iterative removal of all dominated strategies, it is a *rationalisable* strategy.

If there is only one rationalisable strategy, it is the *dominant* strategy (e.g. "defect" in the Prisoner's Dilemma game – see below).

Nash Equilibrium

For n players, $(s_1^*,s_2^*,\ldots,s_n^*)$ is a Nash equilibrium (NE) if and only if: $u_i(s_1^*,s_2^*,\ldots,s_i^*,\ldots,s_n^*) \geq u_i(s_1^*,s_2^*,\ldots,s_i,\ldots,s_n^*)$ for all s_i and all i.

All NEs consist of rationalisable strategies, but not all combinations of rationalisable strategies are NEs.

All combinations of dominant strategies are NEs, but not all NEs consist of dominant strategies.

Mixed strategy equilibrium

A mixed strategy is a probability distribution over strategies. I.e. a given player is playing each of his possible strategies with some probability. For example, firm A produces high output with 40% probability and low output with 60%, and firm B similarly mixes these strategies, but in the ratio 70:30. The game may be one-shot, so the point isn't necessarily that the players alternate between strategies.

In this case the NE consists of optimal probability choices by each player *given* the probability choices of other players.

Once you allow for mixed strategies then every (finite) game has at least one NE.

Non-cooperative game

In a cooperative game, the rules permit binding agreements prior to play. (Hence collusion would be possible in a one-shot cooperative game.) In practice, we are usually concerned with games in which this is not possible, i.e. a non-coooperative game.

Hence strategies are often based on assumptions about what other players are likely to do.

Games of "complete information"

• Players' payoffs as functions of other players' moves are common knowledge.
• Each player knows that other players are 'rational' (i.e. payoff-maximising), and knows that they know that *he* is rational.

Static and dynamic games

In a static (or 'one-shot') game, players move simultaneously and only once. In a dynamic game, players either move alternately, or more than once, or both.

Bertrand and Cournot models of oligopoly competition are both static games. In Bertrand competition, both firms simultaneously choose price for the good they both sell. In Cournot competition, both firms simultaneously choose output quantities q_A and q_B, with price P assumed to be that which will clear the market, i.e. such that demand(P) = $q_A + q_B$.

The Stackelberg model (firm A sets its output quantity first; *then* firm B, knowing q_A, sets its output quantity) is a dynamic game.

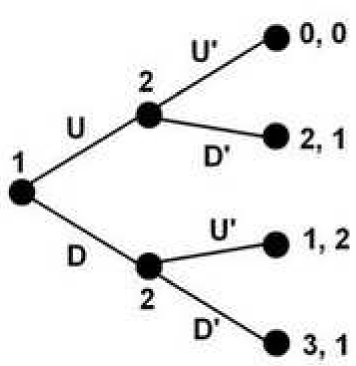
An equilibrium for a dynamic game must satisfy subgame perfection.

Normal and extensive forms

A game expressed in 'normal form' is in the form of a payoff matrix, as shown below for the Prisoner's Dilemma game.

	Cooperate	*Defect*
Cooperate	2, 2	0, 3
Defect	3, 0	1, 1

A game expressed in 'extensive form' shows the 'tree' of the possible move paths depending on what each player does at each stage. A dynamic game can only be shown in extensive form.



Repeated games

A dynamic game in which the same agents repeatedly play a given one-shot game ("form game") in sequence is called a supergame. A supergame can consist of either a finite, or an infinite, repetition of a form game. Or we could have a game with a certain probability *p* of being repeated, i.e. a probability 1 – *p* of breakdown.

A *strategy* in this context can be contingent on what other players have done in previous moves.

An example of a supergame is one in which two firms repeatedly engage in Cournot competition. *Tacit collusion* can be a Nash Equilibrium in this situation, because the strategy "produce low output as long as the other firm did last time" is profit-maximising if the other firm plays the same strategy.

Subgame perfection

For a dynamic game, some Nash equilibria are not acceptable as solutions because one or more players will want to, and be able to, avoid those outcomes. The subgame perfection criterion demands that, at each stage of the game, the strategy followed is still optimal from that point on.

Non-credible threat

A 'non-credible threat' is a strategy that one player is trying to use to manipulate the behaviour of another (usually via the second move in a sequential game), which forms part of a Nash equilibrium but one that is *not* subgame perfect.

The strategy is one which is being claimed in some way by the threatening player (e.g. by means of a signal that he is capable of using it) but which is not credible. Although the threatened player's optimal response to the strategy is to do what the threatening player wants, the former knows that by moving first in a different way, the latter will adopt another strategy to generate a different Nash equilibrium.

In a sense, there is no 'credible threat'. The term 'threat' implies that player 1 *will* do something specifically designed to harm player 2 *if* player 2 doesn't comply, but such a threat would never be carried out in a finite game with perfect information because such a move would not be optimal for player 2. Player 2 will always 'accommodate' when it comes to it.

Entry deterrence

'Entry deterrence' is an example of trying to manipulate a rival player's moves. In this case, it involves an incumbent firm trying to prevent the entry of potential rivals into a market.

Successful entry deterrence depends on avoiding the non-credible threat problem. If you want to make things too difficult for a potential entrant to bother entering, you have to do so in a way which binds you, i.e. you have to commit to a particular strategy. This has to involve *ex ante* (i.e. prior to the other player's move) and *irreversible* action, which prima facie is suboptimal for player 1 (and therefore is said to be 'strategic', i.e. undertaken only for the purposes of affecting the other player's behaviour) but which ultimately pays because it succeeds in deterring entry.

Excess (in the sense of surplus) *capacity* is not an effective way of deterring entry; in fact it represents a non-credible threat. An incumbent would never expand capacity in response to entry, he would always contract. (Unless there is imperfect information, in which case he may try to convince the other player he is irrational.) However, *over-investment* in capacity may succeed in deterring entry. This is the Dixit[1] model in which the incumbent invests irreversibly to expand the capacity at which he can produce at low marginal cost, beyond what he would do left to himself. The point is that this results in a post-entry equilibrium in which his output is higher than what it would have been, and the entrant's output lower — indeed, so low that the latter can't cover its fixed cost.

Moral Hazard

'Moral hazard' arises when player A wishes to contract with player B for the performance of a variable task by B, the outcome of which will depend partly on (i) B's effort and partly on (ii) random factors, and where it is impossible to ascertain how much the outcome is due to (i) versus (ii). The problem is that B does not have as much incentive to perform as would be optimal. In the case of theft insurance, for example, the insured does not have the ideal level of incentive to protect his property because the insurer cannot monitor what he does, and the insured will therefore tend to under-protect it.

There is a connection between credible threats and moral hazard. To avoid moral hazard, A wants B to believe there will be penalties for indulging in 'immoral' behaviour. However, the threat to penalise errant behaviour has to be credible. Either the penalty has to be unavoidable, e.g. criminal legal sanctions, or it has to be somehow in A's interests to apply it. The problem is that the application of a punishment is not usually intrinsically beneficial for the punisher. One possible way out is through reputation: if A's reputation for truth-telling and toughness is valuable to A, then A announcing publicly that a penalty will be imposed could lead to a cost for A if he then fails to implement. In this way, the threat to punish would become credible.

Applying this to the Bank of England, a threat *not to bail out banks in trouble except in very limited circumstances* is at risk of not being credible and therefore of not being effectual, unless reneging on the threat can be regarded as somehow costly for the Bank. However, it is not clear how the Bank, or any of its agents, could suffer from the failure to penalise an errant lender. Possibly when the Bank was still relatively controlled by the government (pre-1997), the desire of the ruling party to be re-elected could have provided such an incentive.

When there is *imperfect information* about whether the failure to carry out a threat is costly for the threatener, it is possible for the threat to be credible by exploiting uncertainty. However, once a player has reneged on his threat without obvious negative repercussions, the possibility of future credible threats is more or less eliminated.

These considerations became particularly relevant in 2007, when the British bank Northern Rock encountered serious liquidity problems, which led to a run on the bank. Although the Bank of England initially attempted to take a tough stance on the matter, it eventually announced that it was guaranteeing investors' deposits, thus undermining its earlier reputation.[2]

NOTES

[1] Dixit, A. (1980) 'The Role of Investment in Entry Deterrence', *Economic Journal* 90, 95-106.

[2] See for example 'Blinking in the face of trouble', *The Economist*, 18 September 2007.

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